

Extrapolation of random fields via level sets

Abstract

The literature on the inter- or extrapolation as well as prediction of random processes and fields is huge. In the infinite variance case, however, the approaches are tailored to specific classes of processes or fields under consideration and the general framework for the extrapolation of heavy-tailed random fields is still missing. We try to fill this gap by noting that two random fields are, in a sense, similar if their level (or excursion) sets are similar. To be more precise, two random fields modeling some feature with the same structure of excursions have the same total amount of this feature exceeding each level over a fixed time interval or a spatial domain. This is certainly of interest for practical applications to insurance (with the feature being the claim size), environmetrics (e.g. for the amount of environmental pollution or radiation), etc. In our approach, similarity is measured by the expected volume of the symmetric difference of the level sets. It is sometimes also called expected distance in measure.

We use this concept of excursion sets for the extrapolation of stationary random fields. Doing so, we define excursion sets for the field and its linear predictor, and then minimize the expected volume of the symmetric difference of these sets under the condition that the univariate distributions of the predictor and of the field itself coincide. We illustrate the new approach on Gaussian random fields.

Keywords

Stationary random field, Gaussian random field, Extrapolation, Linear prediction, Excursion, Level set.

References: