Approximate Reference Priors for Gaussian Random Fields

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Abstract

When modeling spatially correlated data using Gaussian random fields, exact reference priors for the model parameters have been recommended for objective Bayesian analysis. But their use in practice is hindered by its complex formulation and the associated computational costs. In this work, we propose a new class of default prior distributions for the parameters of Gaussian random fields that approximate exact reference priors. It is based on the spectral representation of stationary random fields and their spectral density functions. These approximate reference priors maintain the major theoretical advantages of exact reference priors, but at a much lower computational cost. Unlike the situation for exact reference priors, we show that the marginal prior of the range parameter in the Matern correlation family is always proper, regardless of the mean function or degree of smoothness of the correlation function, and also establish the propriety of the joint reference posterior of the model parameters. Finally, an illustration is provided with a spatial data set of lead pollution in Galicia, Spain.

Keywords

Bayesian analysis, Default prior, Geostatistics, Spectral representation.

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