A robust scalar-on-function partial quantile regression

Mujgan Tez¹, Ufuk Beyaztas¹ and Han Lin Shang²

¹Department of Statistics, Marmara University, Kadikoy, Istanbul, Turkey ²Department of Actuarial Studies and Business Analytics, Macquarie University, Sydney, NSW 2109, Australia

Abstract

Scalar-on-function quantile regression is a powerful regression model to characterize the entire conditional distribution of a scalar response variable for a given functional predictor. Compared with the conditional mean regression-based scalar-on-function regression model, the scalar-on-function quantile regression is robust to outliers in the response variable. However, it is susceptible to outliers in the functional predictor (called leverage points). This is because the influence function of the regression quantiles is bounded in the response variable but unbounded in the predictor space. The leverage points may alter the eigenstructure of the predictor matrix, leading to poor estimation and prediction results. This study proposes a robust scalar-onfunction quantile regression method to robustly estimate the model parameters and produce reliable predictions in the presence of both outliers and leverage points. The proposed method is based on a functional partial quantile regression procedure. We propose a robust partial quantile covariance to obtain functional partial quantile components of the scalar-on-function regression model. In the functional partial quantile decomposition, the robustness is obtained by iteratively reweighting the functional predictor. After such a decomposition, the infinite-dimensional scalar-on-function quantile regression model is approximated in the finite-dimensional space by a quantile regression of the scalar response on the partial quantile scores. The estimation and prediction performance of the proposed method is evaluated by a series of Monte-Carlo experiments and an empirical data example, and the results are compared favorably with several existing methods.

Keywords

Functional data, Iteratively reweighting, Partial quantile covariance, Robust estimation.