

# Skewness and Kurtosis of Mean-Variance Normal Mixtures

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## Abstract

Mean-variance mixtures of normal distributions are very flexible distributions. They can model many nonnormal features, as for example skewness, kurtosis, or multimodality. Special cases include mixtures of two normal distributions with proportional covariance matrices and scale mixtures of normal distributions. This paper investigates the skewness and the kurtosis of multivariate mean-variance normal mixtures. Firstly, it derives the analytical forms of best-known measures of multivariate skewness and kurtosis, thus highlighting some of their shortcomings. Secondly, it applies these results to portfolio optimization, model-based clustering, invariant coordinate selection, normalizing linear transformations, projection pursuit and normality testing. The practical relevance of theoretical results in the paper is assessed using both real and simulated datasets.

## Keywords

Kurtosis, mixture model, projection pursuit, skewness.

## References:

- Lee, S.X. and McLachlan, G.J. (2019). On mean and/or variance mixtures of normal distributions. In Scientific Meeting of the Classification and Data Analysis Group of the Italian Statistical Society, 117-127. Cham: Springer.
- Malkovich, J.F. and Afifi, A.A. (1973). On Tests for Multivariate Normality. *J. Amer. Statist. Ass.* 68, 176-179
- Mardia, K.V. (1970). Measures of multivariate skewness and kurtosis with applications. *Biometrika* 57, 519-530.
- Móri T.F., Rohatgi V.K. and Székely G.J. (1993). On multivariate skewness and kurtosis. *Theory Probab. Appl.* 38, 547-551.