Joining iso-structured models with commutative orthogonal block structure

Carla Santos[1], Cristina Dias[2], Célia Nunes[3], João T. Mexia[4].

[1] *Polytechnic Institute of Beja, Beja, and Center for Mathematics and Applications, New University of Lisbon, Lisbon, Portugal, carla.santos@ipbeja.pt*

[2] *Polytechnic Institute of Portalegre, Portalegre, and Center for Mathematics and Applications, New University of Lisbon, Lisbon, Portugal, cpsd@ipportalegre.pt*

[3] *Department of Mathematics and Center of Mathematics and Applications, University of Beira Interior, Covilhã, Portugal, celian@ubi.pt*

[4] *Department of Mathematics and Center for Mathematics and Applications, Nova University of Lisbon, Lisbon, Portugal, jtm@fct.unl.pt*

Abstract

A model with commutative orthogonal block structure (COBS) is a linear mixed model whose variance-covariance matrix is a linear combination of known pairwise orthogonal projection matrices that add up to the identity matrix, and commutes with the orthogonal projection matrix on the space spanned by the mean vector. COBS, as particular class of the models with orthogonal block structure, arose in order to obtain optimal estimation for variance components of blocks and contrasts of treatments. Resorting to their algebraic structure, we study COBS and the operation of models joining. Since joining COBS originates a new COBS, ensuring that the conditions for the good properties of the estimators are preserved, we explore performing the operation of models joining with iso-stuctured COBS, that is, with COBS with identical space spanned by their mean vectors and having covariance matrices that are linear combinations of the same pairwise orthogonal projection matrices.

Keywords

Best linear unbiased estimator, Mixed model, Jordan algebra, Operation with models.

References

Jordan, P., Von Neumann, J., Wigner, E. (1934) On an algebraic generalization of the quantum mechanical formulation, *Annals of Math.* v. 35, n.1.

Nelder, J.A. (1965a) The analysis of randomized experiments with orthogonal block structure I. Block structure and the null analysis of variance, *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.*, v. 283, 147–162.

Nelder, J.A. (1965b) The analysis of randomized experiments with orthogonal block structure II. Treatment structure and the general analysis of variance, *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.*, v. 283, 163–178.

Santos, C., Nunes, C., Dias, C., Mexia, J. T. (2017) Joining models with commutative orthogonal block structure. *Linear Algebra and its Applications*, v. 517, 235–245.

Zmyślony, R. (1978) A characterization of best linear unbiased estimators in the general linear model, *Lecture Notes in Statistics*, v. 2, 365–373.

Acknowledgments This work was partially supported by national founds of FCT-Fundação para a Ciência e Tecnologia (Foundation for Science and Technology), Portugal, under UIDB/00297/2020 and UIDB/00212/2020.