

Large deviations of extremal eigenvalues of sample covariance matrices

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Abstract

Large deviations of the largest and smallest eigenvalues of $\mathbf{X}\mathbf{X}^T/n$ are studied in this note, where $\mathbf{X}_{p \times n}$ is a $p \times n$ random matrix with independent and identically distributed (i.i.d.) sub-Gaussian entries. The assumption imposed on the dimension size p and the sample size n is $p = p(n) \rightarrow \infty$ with $p(n) = o(n)$. This study generalizes one result obtained in [?].

Keywords and phrases: Large deviations; sample covariance matrices; extremal eigenvalues

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