Estimation of linearly structured block covariance matrix

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Abstract

In this paper, the study of the relation between two groups of characteristics observed on sample units is of interest. The observation received from the experimental unit has a form of vector and it can be divided into two subvectors, corresponding with two group of characteristics. The relation between features is expressed in cross-covariance matrix, which structure is considered. The covariance structure with symmetric positive definite diagonal blocks and linearly structured off-diagonal blocks is assumed. Moreover, the covariance matrix with all blocks linearly structured is studied. The aim of the research is to determine structured estimator of the covariance matrix with good statistical properties (such as consistency), which is both (i) positive definite and (ii) well-conditioned. In the case of structured covariance matrix the maximum likelihood estimation procedure can provide to complex numerical problem. Alternative approach is proposed, which is based on least squares method ([Ohlson and von Rosen (2010)]), shrinkage method ([Ledoit and Wolf (2004)]), and additional algebraic improvement. New approach, called quasi shrinkage estimation, is much faster and it allows to determine estimators in simple form, with good statistical properties. The obtained estimator has a similar form as the estimators given in [Markiewicz and Mieldzioc (2022)]. Simulations study are performed to compare maximum likelihood estimators and quasi shrinkage estimators of structured covariance matrices by some criteria. Furthermore, characteristics of quadratic spaces of block matrices are derived.

Keywords

Block covariance matrix, Least squares method, Linear structure, Quadratic space Shrinkage method.

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